A robust tool for localizing eigenvalues *

Bernard Philippe † Emmanuel Kamgnia ‡

The localization of some eigenvalues of a given matrix in a domain of the complex plane is of interest in scientific applications. When the matrix is real symmetric or complex hermitian, a procedure based on computations of Sturm sequences allows to safely apply bisections on real intervals to localize the eigenvalues. The problem is much harder for real non symmetric or complex non hermitian matrices and especially for non normal ones. Our work deals with this last case.

For taking into account, possible perturbations of the matrix, Godunov [2] and Trefethen [8] have separately defined the notion of the of ϵ-spectrum or pseudospectrum of a matrix $A \in \mathbb{R}^{n\times n}$ to address the problem. The problem can then be reformulated as that of determining level curves of the 2-norm of the resolvent $R(z) = (zI - A)^{-1}$. A pseudospectrum determines an enclosure for some eigenvalues.

A dual approach can be considered: given some curve (Γ) in the complex plane, count the number of eigenvalues of the matrix A that are surrounded by (Γ). The number of surrounded eigenvalues is determined by evaluating the integral $\frac{1}{2i\pi}\int_\Gamma \frac{d}{dz}\log\det(zI - A)dz$. This problem was considered in [1] where several procedures were proposed and more recently in [4] where the stepsize control in the quadrature is deeply studied.

Our present goal is to combine the two approaches. For a large sparse matrix A, we propose to first consider the method PAT [6] which is a path following method to determine a level curve of the function $s(z) = \sigma_{\min}(zI-A)$. We then discuss how to insert the method EIGENCNT of [4] for computing the number of eigenvalues included in a pseudospectrum obtained by PAT or by its parallel version PPAT [5]. The combined procedure will be based on a computing kernel which provides the two numbers $(\sigma_{\min}(zI - A), \det(zI - A))$ for any complex number $z \in \mathbb{C}$. These two numbers can be obtained through a common LU factorization of $(zI - A)$. In order to obtain a second level of parallelism, we consider a preliminary transformation similar to the approach developed in SPIKE [7, 3].

References

[1] O. Bertrand and B. Philippe, *Counting the eigenvalues surrounded by a closed curve*, Siberian Journal of Industrial Mathematics, 4 (2001), pp. 73–94.

[2] S. Godunov, *Spectral portraits of matrices and criteria of spectrum dichotomy*, in Third Int'l. IMACS-CAMM Symposium on Computer Arithmetic and Enclosure Methods, L. Atanassova and J. Hezberger, eds., Amsterdam, 1992.

[3] E. Kamgnia, L. B. Nguenang, and B. Philippe, *Some efficient methods for computing the determinants of large sparse matrices*, in 11th African Conference on Research in Computer Science and Applied Mathematics (CARI'2012), Algiers, 2012.

[4] E. Kamgnia and B. Philippe, *Counting eigenvalues in domains of the complex field*, Electron. Trans. Numer. Anal., Accepted for publication (2012). An early version is available at http://hal.archives-ouvertes.fr/hal-00634065/PDF/RR-7770.pdf.

[5] D. Mezher and B. Philippe, *Parallel computation of pseudospectra of large sparse matrices*, Parallel Comput., 28 (2002), pp. 199–221.

*This research is partly supported by LIRIMA, http://lirima.org/, team MOMAPPLI.

†INRIA, Campus Beaulieu, 35042 Rennes Cedex, France; mail: Bernard.Philippe@inria.fr

‡University of Yaounde I, Cameroon; mail: erkamgnia@yahoo.fr

[6] ——, *PAT: A reliable path following algorithm*, Numer. Algorithms, 29 (2002), pp. 131–152.

[7] E. Polizzi and A. Sameh, *A parallel hybrid banded systems solver: the SPIKE algorithm*, Par. Comp., 32 (2006), pp. 177–194.

[8] L. Trefethen, *Pseudospectra of matrices*, in Numerical Analysis, D. F. Griffiths and G. A. Watson, eds., Longman, 1992, pp. 234–266.